Actuarial Geometry: Volumetric and Temporal Diversification of Insurance Risk

Stephen J. Mildenhall
University of Wisconsin, Madison

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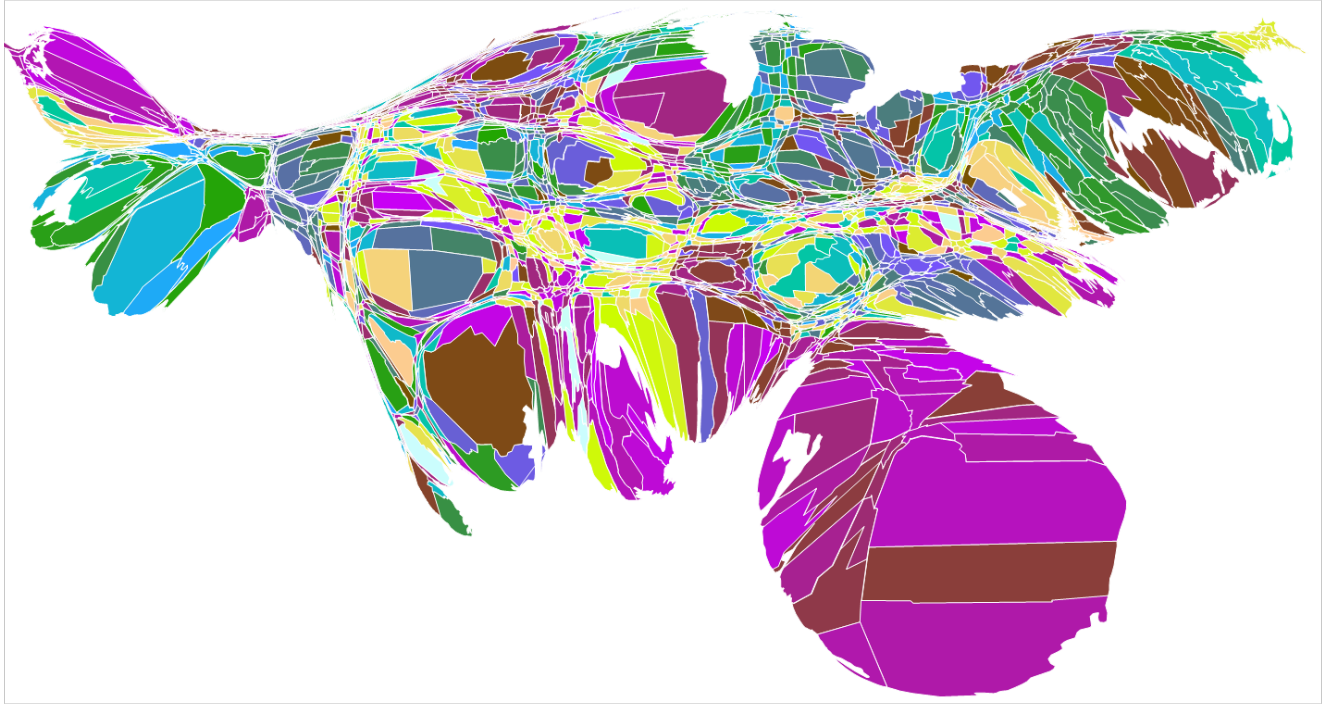

Outline

1. Insurance pricing frameworks
2. Insurance risk is not volumetrically diversifying
3. Insurance losses are not homogeneous with respect to volume
4. Homogeneous model is not even “locally” appropriate
5. Empirical data and supporting evidence
6. Four models based on Levy processes
7. Why bother with general Levy processes vs. compound Poisson processes?
8. So what? Can we see impact in prices?
9. Observed correlations and copulas
10. How the results are used

1. Insurance Pricing Frameworks

	Risk Theory	Finance	Actuarial
1900s	Bachelier		Bureau rates
1930s	Cramer-Lundberg Esscher		Bureau rates Bureau rates
1950s	Levy, Kolmogorov, Khintchine, Ito		…
1960s		Portfolio theory CAPM	Bureau rates Bailey investment inc.
1970s	Buhlmann Borch	Systemic vs. diversifiable risk	Ferrari, ROE 1978 ind. u/w profit
1980s		Option pricing, no arbitrage, comparables	
1990s	Artzner et al. coherent measure of risk		Cat Models
2000s	Wang transform	Phillips, Cummins, Allen Myers-Read Froot et al.	Idiosyncratic risk matters (Froot 2001)
2010s	Levy processes, optimal dividends	Zanjani	2004 ind. u/w profit Debt vs. equity

2. Insurance Risk is not Volumetrically Diversifying

- Expected Loss ($) = Volume ($ / t) x Time (t)
- A(x,t) := random variable representing aggregate losses from volume x insured for time t
  - E[A(x,t)] = xt = expected loss
- Insurance risk is not volumetrically diversifying, meaning
  - CV(A(x,t)) does not tend to zero as x increases, for fixed t
  - Recall coefficient of variation = CV = standard deviation / mean
- Practical meaning
  - It is impossible to diversify away all insurance risk by growing larger
- How to investigate?
  - CV(A) = CV(A / p) = CV(loss ratio), p = fixed premium
  - Look at volatility in loss ratio with volume
  - Premium (and company) effects can be removed using an ANOVA; does not change conclusions
- Data source: NAIC Annual Statement, Schedule P
  - Gross, ultimate loss ratios with 10 accident year history for most lines
  - Major lines: WC, Commercial Auto, HO, PPA, CMP, Other Liability etc.

Annual Statement for the year 2005 of the **CONTINENTAL CASUALTY COMPANY**

SCHEDULE P - PART 1D - WORKERS' COMPENSATION

($000 omitted)

Years in Which Premiums Were Earned and Losses Were Incurred	Premiums Earned 1 Direct and Assumed	2 Ceded	3 Net (Cols. 1 - 2)	Loss and Loss Expense Payments: Loss Payments 4 Direct and Assumed	5 Ceded	Defense and Cost Containment Payments 6 Direct and Assumed	7 Ceded	Adjusting and Other Payments 8 Direct and Assumed	9 Ceded	10 Salvage and Subrogation Received	11 Total Net Paid (Cols. 4 - 5 + 6 - 7 + 8 - 9)	12 Number of Claims Reported-Direct and Assumed
1. Prior	XXX	XXX	XXX	156,422	7,531	10,365	72	3,530	(10)	33,847	162,724	XXX
2. 1996	1,746,768	9,914	1,736,854	859,568	12,096	66,293	1,254	93,376	8	51,572	1,005,879	348,154
3. 1997	1,342,521	(151,161)	1,493,682	1,012,510	11,854	83,723	1,705	73,653	0	60,094	1,156,327	384,917
4. 1998	1,704,209	28,043	1,676,166	1,303,449	(17,438)	110,787	2,411	94,356	5	66,457	1,523,614	423,447
5. 1999	1,723,216	270,103	1,453,113	1,409,971	413,039	115,987	12,402	81,396	9	64,051	1,181,904	424,836
6. 2000	1,390,797	194,283	1,196,514	1,104,815	412,884	103,020	14,787	45,466	4	54,749	825,625	357,680
7. 2001	1,037,840	583,732	454,108	888,300	411,142	80,207	13,631	65,486	24	39,772	609,196	292,642
8. 2002	1,464,414	180,605	1,283,809	583,945	47,368	56,075	2,123	83,137	0	27,115	673,665	226,035
9. 2003	1,517,227	426,236	1,090,991	436,436	44,729	40,689	1,652	63,550	0	12,505	494,294	158,810
10. 2004	1,504,575	208,397	1,296,178	274,586	32,821	24,354	1,159	34,879	0	4,999	299,839	131,659
11. 2005	1,173,428	205,268	968,160	89,976	4,481	5,991	201	29,498	(115)	505	120,898	82,681
12. Totals	XXX	XXX	XXX	8,119,978	1,380,507	697,490	51,398	668,328	(75)	415,666	8,053,965	XXX

	Losses Unpaid: Case Basis 13 Direct and Assumed	14 Ceded	Bulk + IBNR 15 Direct and Assumed	16 Ceded	Defense and Cost Containment Unpaid: Case Basis 17 Direct and Assumed	18 Ceded	Bulk + IBNR 19 Direct and Assumed	20 Ceded	Adjusting and Other Unpaid 21 Direct and Assumed	22 Ceded	23 Salvage and Subrogation Anticipated	24 Total Net Losses and Expenses Unpaid	25 Number of Claims Outstanding-Direct and Assumed
1. Prior	1,466,509	101,202	386,407	171,778	0	0	61,370	124	47,577	(411)	41,783	1,689,170	11,556
2. 1996	107,654	1,197	13,695	(100)	0	0	11,487	20	4,376	0	868	136,095	895
3. 1997	157,495	1,506	24,545	2,825	0	0	20,896	20	4,728	0	1,225	203,313	1,204
4. 1998	230,749	3,959	42,455	83	0	0	18,732	118	6,869	(141)	2,057	294,786	1,939
5. 1999	297,512	78,852	36,042	13,571	0	0	7,161	144	6,207	0	3,303	254,355	2,833
6. 2000	275,517	99,191	58,059	27,994	0	0	23,111	465	6,721	0	1,229	235,758	3,172
7. 2001	289,330	424,745	124,500	84,165	0	0	20,449	492	6,548	0	151	(68,575)	3,067
8. 2002	188,949	16,288	130,920	20,025	0	0	37,282	926	8,618	0	16,653	328,530	2,494
9. 2003	181,644	24,726	171,086	18,836	0	0	36,611	904	9,118	0	16,486	353,993	3,095
10. 2004	181,275	24,505	320,179	71,518	0	0	45,246	3,162	11,366	0	20,379	458,881	4,783
11. 2005	147,550	8,236	402,348	85,228	0	0	49,821	2,236	71,660	0	21,843	575,679	13,435
12. Totals	3,524,184	784,407	1,710,236	495,923	0	0	332,166	8,611	183,788	(552)	125,977	4,461,985	48,473

	Total Losses and Loss Expenses Incurred 26 Direct and Assumed	27 Ceded	28 Net	Loss and Loss Expense Percentage (Incurred/Premiums Earned) 29 Direct and Assumed	30 Ceded	31 Net	Nontabular Discount 32 Loss	33 Loss Expense	34 Inter-Company Pooling Participation Percentage	Net Balance Sheet Reserves after Discount 35 Losses Unpaid	36 Loss Expenses Unpaid
1. Prior	XXX	XXX	XXX	XXX	XXX	XXX	0	0	XXX	1,579,936	109,234
2. 1996	1,156,449	14,475	1,141,974	66.2	146.0	65.7	0	0	0.00	120,252	15,843
3. 1997	1,377,550	17,910	1,359,640	102.6	(11.8)	91.0	0	0	0.00	177,709	25,604
4. 1998	1,807,397	(11,003)	1,818,400	106.1	(39.2)	108.5	0	0	0.00	269,162	25,624
5. 1999	1,954,276	518,017	1,436,259	113.4	191.8	98.8	0	0	0.00	241,131	13,224
6. 2000	1,616,709	555,325	1,061,383	116.2	285.8	88.7	0	0	0.00	206,391	29,367
7. 2001	1,474,820	934,199	540,621	142.1	160.0	119.1	0	0	0.00	(95,080)	26,505
8. 2002	1,088,925	86,730	1,002,195	74.4	48.0	78.1	0	0	0.00	283,556	44,974
9. 2003	939,134	90,847	848,287	61.9	21.3	77.8	0	0	0.00	309,168	44,825
10. 2004	891,885	133,165	758,720	59.3	63.9	58.5	0	0	0.00	405,431	53,450
11. 2005	796,844	100,267	696,577	67.9	48.8	71.9	0	0	0.00	456,434	119,245
12. Totals	XXX	XXX	XXX	XXX	XXX	XXX	0	0	XXX	3,954,090	507,895

Years in Which Premiums Were Earned and Losses Were Incurred	Premiums Earned 1 Direct and Assumed	2 Ceded	3 Net (Cols. 1 - 2)
1. Prior	XXX	XXX	XXX
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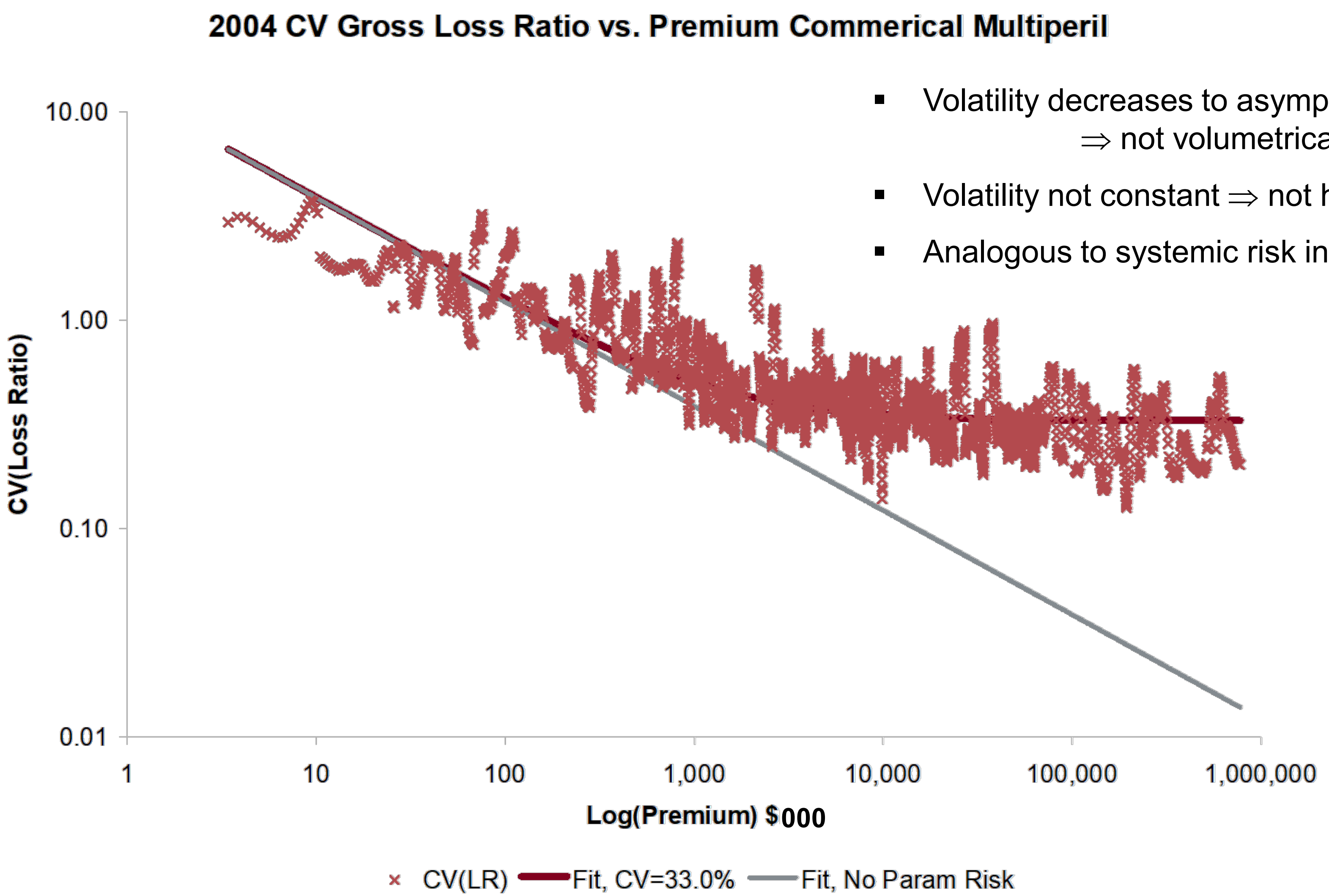
10. 2004	1,504,575	208,397	1,296,178
11. 2005	1,173,428	205,268	968,160
12. Totals	XXX	XXX	XXX

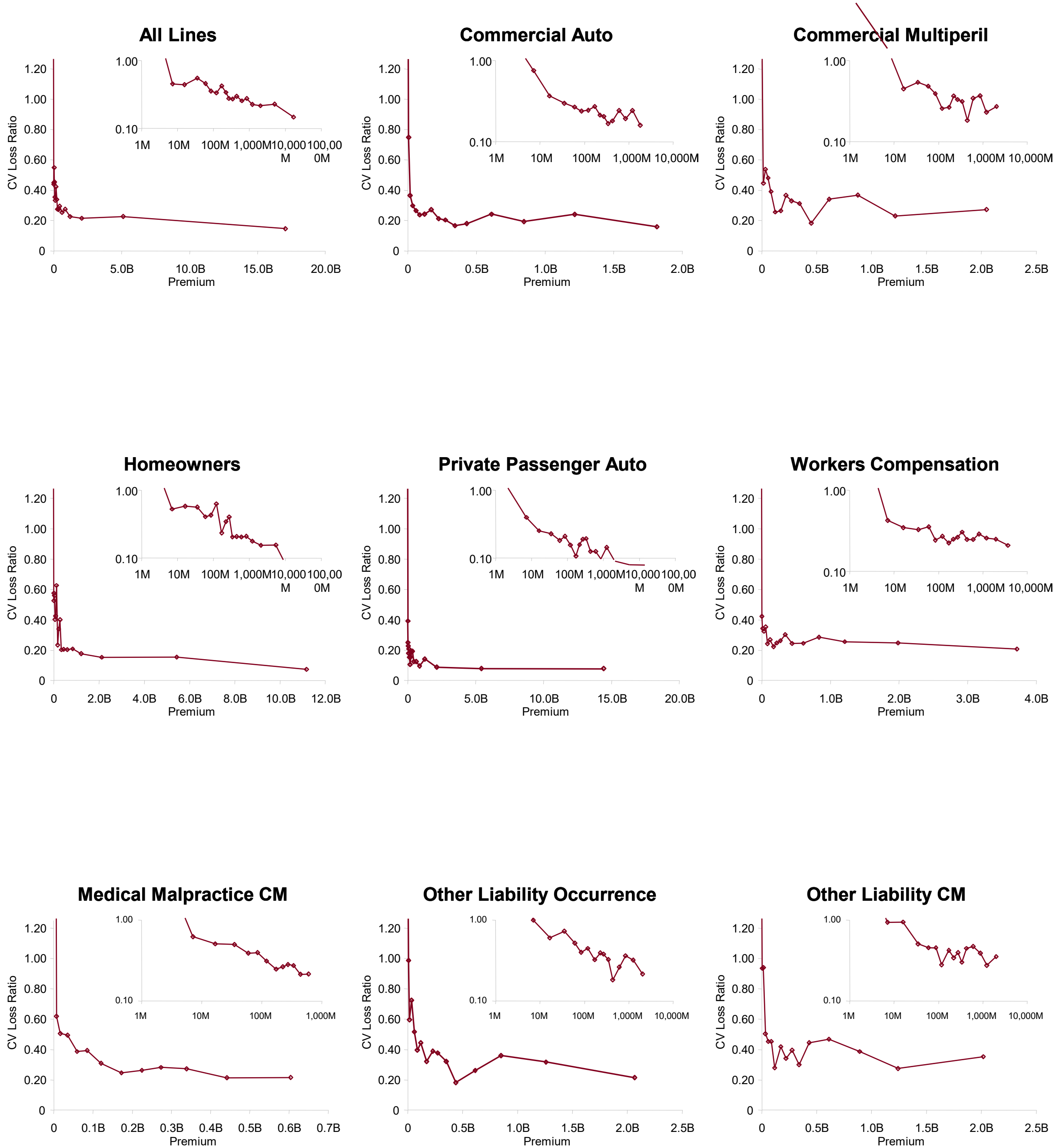
Loss and Loss Expense Percentage (Incurred/Premiums Earned) 29 Direct and Assumed	30 Ceded	31 Net
XXX	XXX	XXX
66.2	146.0	65.7
102.6	(11.8)	91.0
106.1	(39.2)	108.5
113.4	191.8	98.8
116.2	285.8	88.7
142.1	160.0	119.1
74.4	48.0	78.1
61.9	21.3	77.8
59.3	63.9	58.5
67.9	48.8	71.9
XXX	XXX	XXX

2. Risk is not Volumetrically Diversifying



- Volatility decreases to asymptote > 0
  ⇒ not volumetrically diversifying
- Volatility not constant ⇒ not homogeneous
- Analogous to systemic risk in stock portfolio

- Asymptote is a risk characteristic for each line
- It varies substantially across lines
- It is reasonably constant over time
- Homeowners



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3. Insurance Losses are not Homogeneous with Respect to Volume

- **Homogeneous** model: $A(x,t) = xR_t$
  - R_t a “return” random variable independent of volume x
  - For assets x is position size and R_t is return or unit price
  - Introduces a natural vector space structure for assets, with basis the return vectors $R_{i,t}$

- Homogeneity implies
  - Shape of aggregate loss distribution independent of volume
  - No volume based diversification
  - $A(x,t)$ has constant coefficient of variation (volatility) with x

- Homogeneous models are not appropriate for insurance
  - Consider probability of zero losses: $Pr(xX=0) = Pr(X=0)$
  - Implies the probability of observing a zero loss is **independent of volume x**

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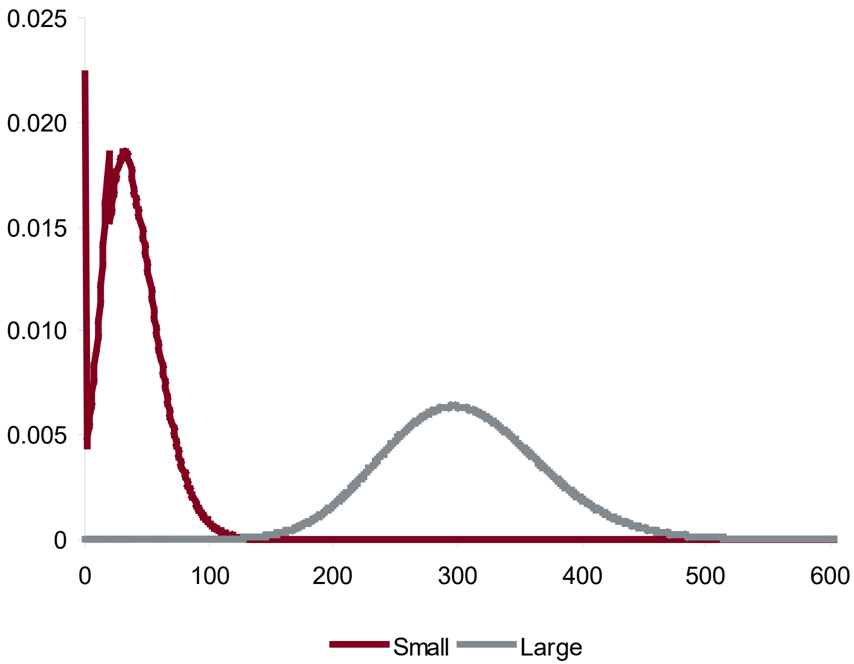
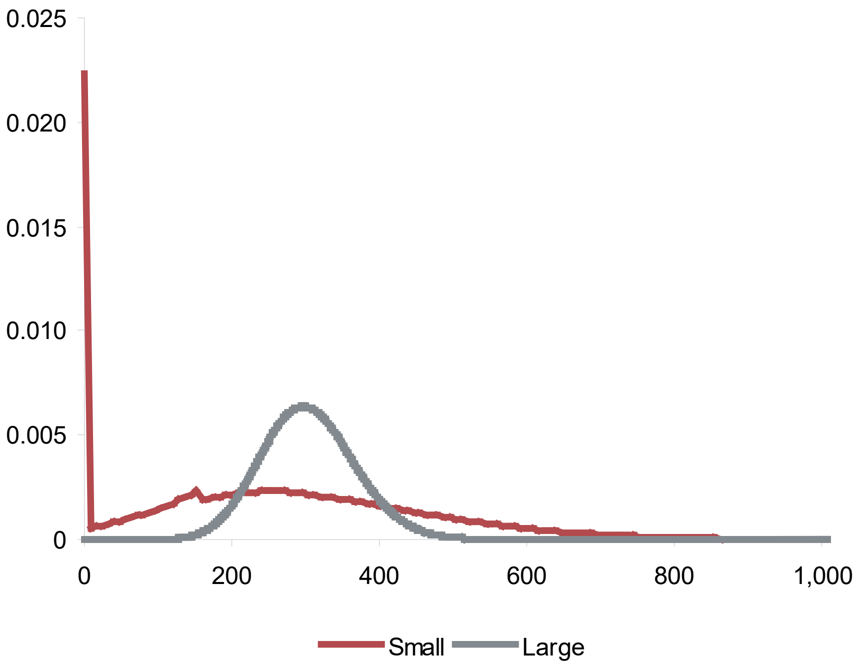
3. Insurance Losses are not Homogeneous with Respect to Volume





- Consider probability of zero claims in small and large books
- Compound Poisson aggregate losses, average severity 10
  - Small: claim count 4
  - Large: claim count 32
- Left plot un-scaled; right plot scaled
- Homogeneous distributions would be indistinguishable in scaled plot
  - Note decrease in variance on right hand plot
- Matlab code: ifft(exp(4 * (fft(severity) – 1)))

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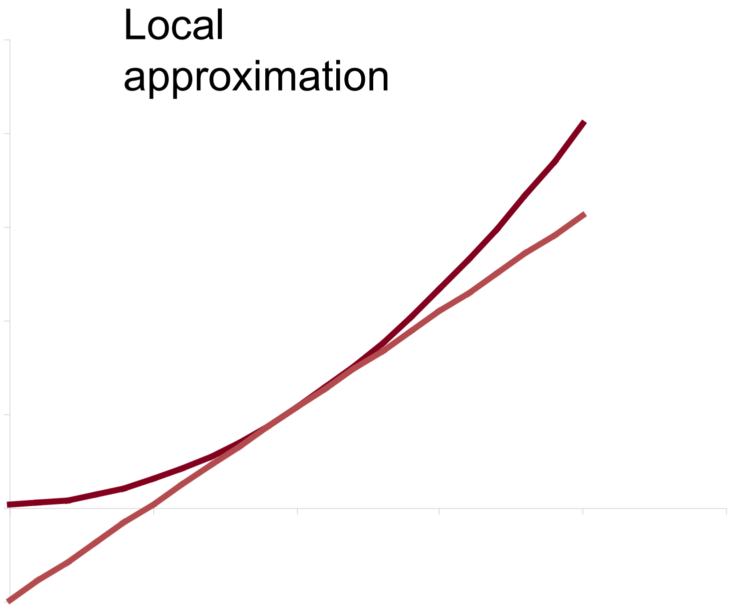
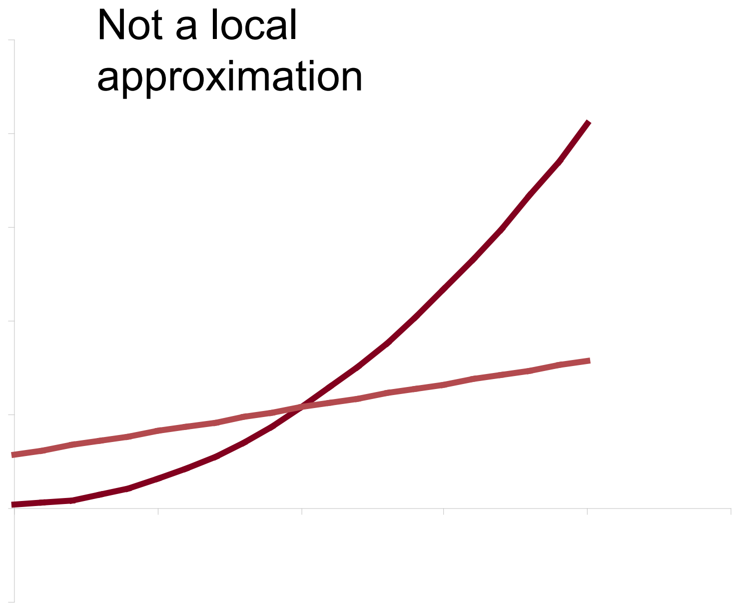
3. Insurance Losses are not Homogeneous with Respect to Volume

- Geometric Brownian motion model is homogeneous wrt volume
  - $S_t = S_0 \exp((\mu - \sigma^2/2)\, t + \sigma B_t)$, where B_t is a Brownian motion
  - Volume = S_0
  - Return = $\exp((\mu - \sigma^2/2)\, t + \sigma B_t)$
  - It is not homogeneous wrt to time t

4. Homogeneity is not “Locally” Appropriate





- Consider tX_1 as a homogeneous approximation to a process X_t, agreeing at t=1
- Local approximation: one holding to first order in a neighborhood of a point
  - First-order equality required by any theory considering derivatives or marginal impacts
    - Myers-Read and gradient based methods of capital allocation
  - Equality at a point does not imply first order approximation
- Requires notion of **derivative** which requires a **direction**

4. Homogeneity is not "Locally" Appropriate

- Recall the time/volume symmetry
  - $E[\, A(x,t)\,] = xt$ = expected loss

  and to be consistent with stochastic process literature assume volume $x=1$ is fixed and let t proxy volume or time

- Model losses $X(t)$ as a (mixed) compound Poisson distribution and suppose
  - Expected claim count = t
  - E(severity) = 1
  - So E(X(t)) = E(severity) x E(claim count) = t

- A homogeneous approximation to the family $X(t)$ near $t = 1$ is given by $t\,X(1)$

- We will show this is **not** a local approximation

- Have two maps from $[0,\infty) \rightarrow \{\text{ risks }\}$, agreeing at $t = 1$:
  - $m(t) = X(t)$, compound Poisson claim count t, (Glenn) Meyers embedding; not homogeneous
  - $k(t) = t\,X(1)$, asset or Kalkbrener embedding; homogeneous by construction

4. Homogeneity is not "Locally" Appropriate

- Let $\rho : \{ \text{risks} \} \to \mathbf{R}$ be a real-valued risk measure
  - Standard deviation, downside risk, higher moment, percentile (=Value-at-Risk, VaR), TVaR
  - Tasche, Denault, Fischer, Myers-Read,… show we should be interested in $\partial \rho / \partial t$, the rate of change of ρ with volume, in a given line of business or risk type

- Two compositions $\rho \circ k$, $\rho \circ m$: $[0,\infty) \to \{ \text{risks} \} \to \mathbf{R}$ both give single valued functions of a single real variable t, and we can often easily compute derivatives

- For ρ = standard deviation and severity ≡ 1 (so the compound Poisson is just Poisson) we have
  - $\rho \circ m(t) = \rho(m(t))$ = std dev(Poisson(t)) = $\sqrt{t}$, but
  - $\rho \circ k(t) = \rho(k(t))$ = std dev(t Poisson(1)) = t, and so $\mathbf{\partial (\rho \circ k) / \partial t = 1 \neq \partial (\rho \circ m) / \partial t = t^{-1/2}/2}$

- Looking at the compositions masks the complexity of the embedding "paths" k and m in { risks }

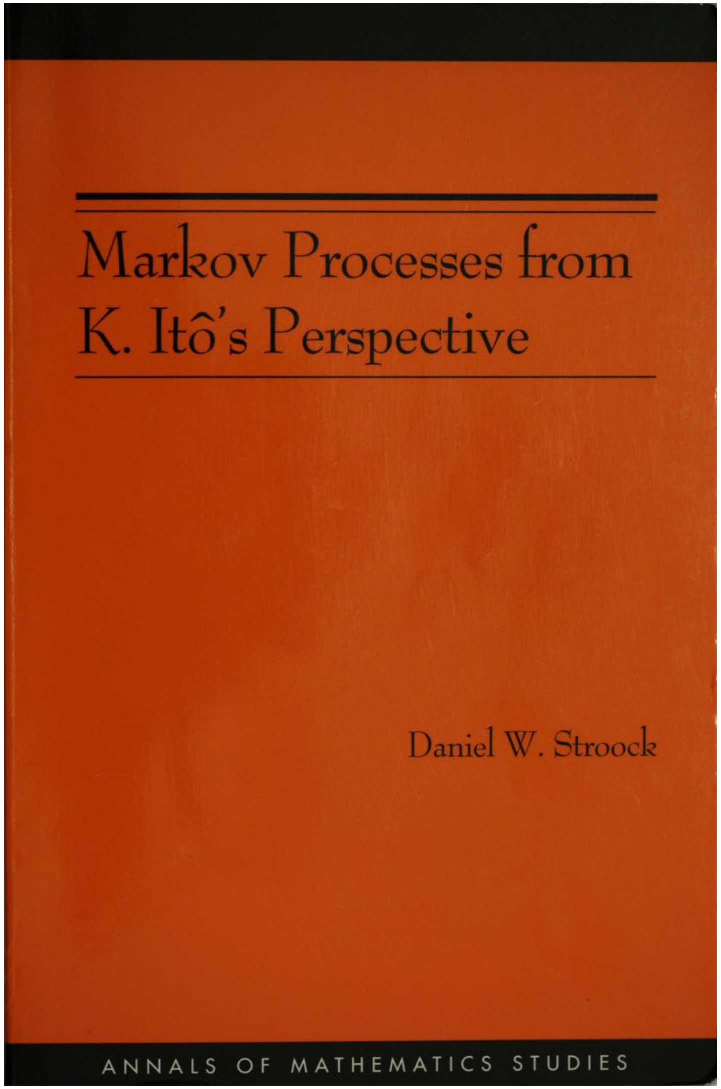
- In terms of derivatives of ρ, "$D\rho_{\text{at a "point"=random variable } X(1)}$(in a direction = k'(1))", example shows

  - $D\rho_{X(1)}(k'(1)) := \partial (\rho \circ k) / \partial t \neq \partial (\rho \circ m) / \partial t =: D\rho_{X(1)}(m'(1))$

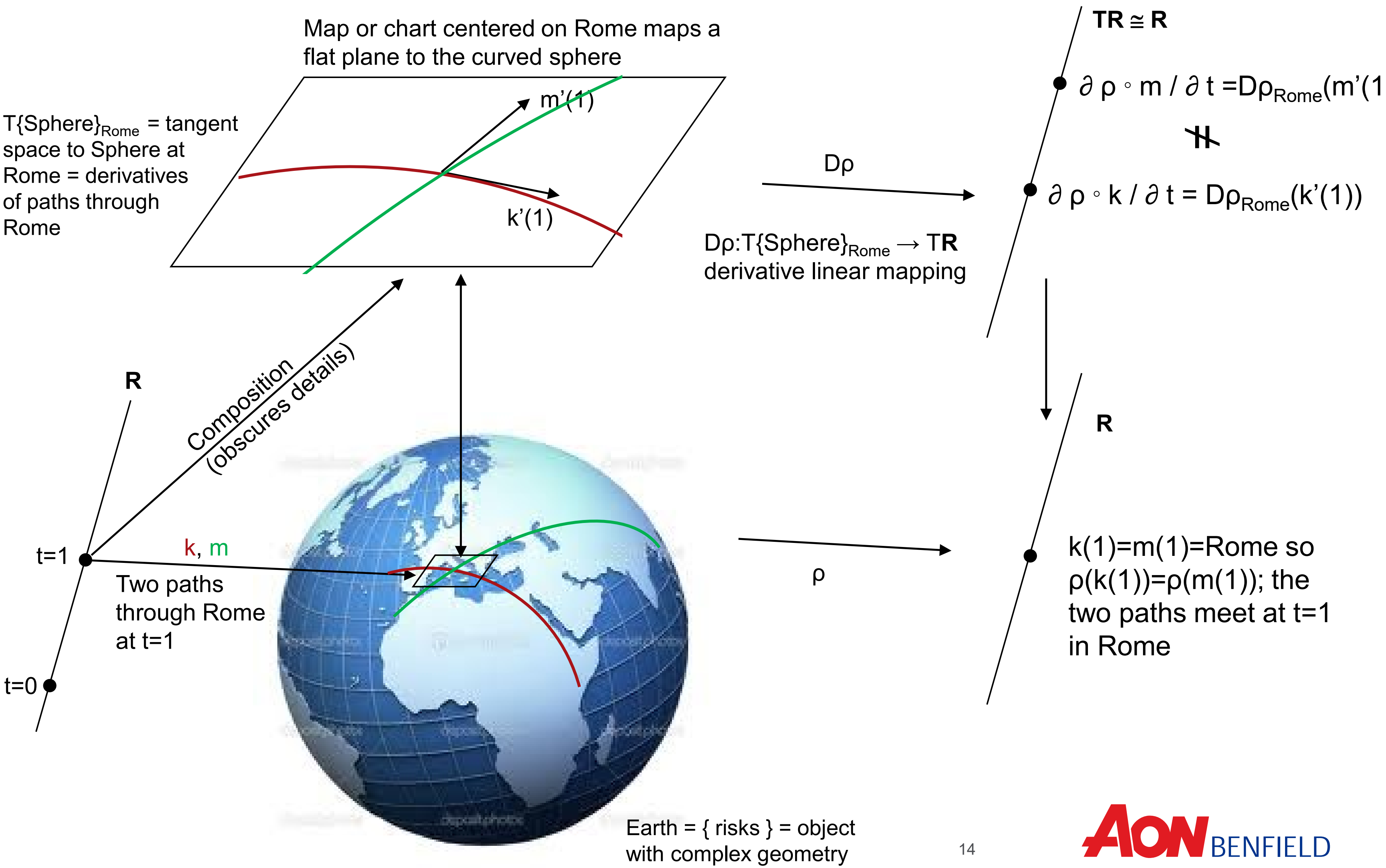
  - $D\rho_{X(1)}(k'(1)) \neq D\rho_{X(1)}(m'(1))$ implies **directions m'(1) ≠ k'(1) are different**

- What are the directions m'(1) and k'(1)? What do these intermediate derivatives mean?
  - In the asset setting there are no mysteries for k: working in a vector space setting

Serendipitous Moment…



4. Homogeneity is not “Locally” Appropriate



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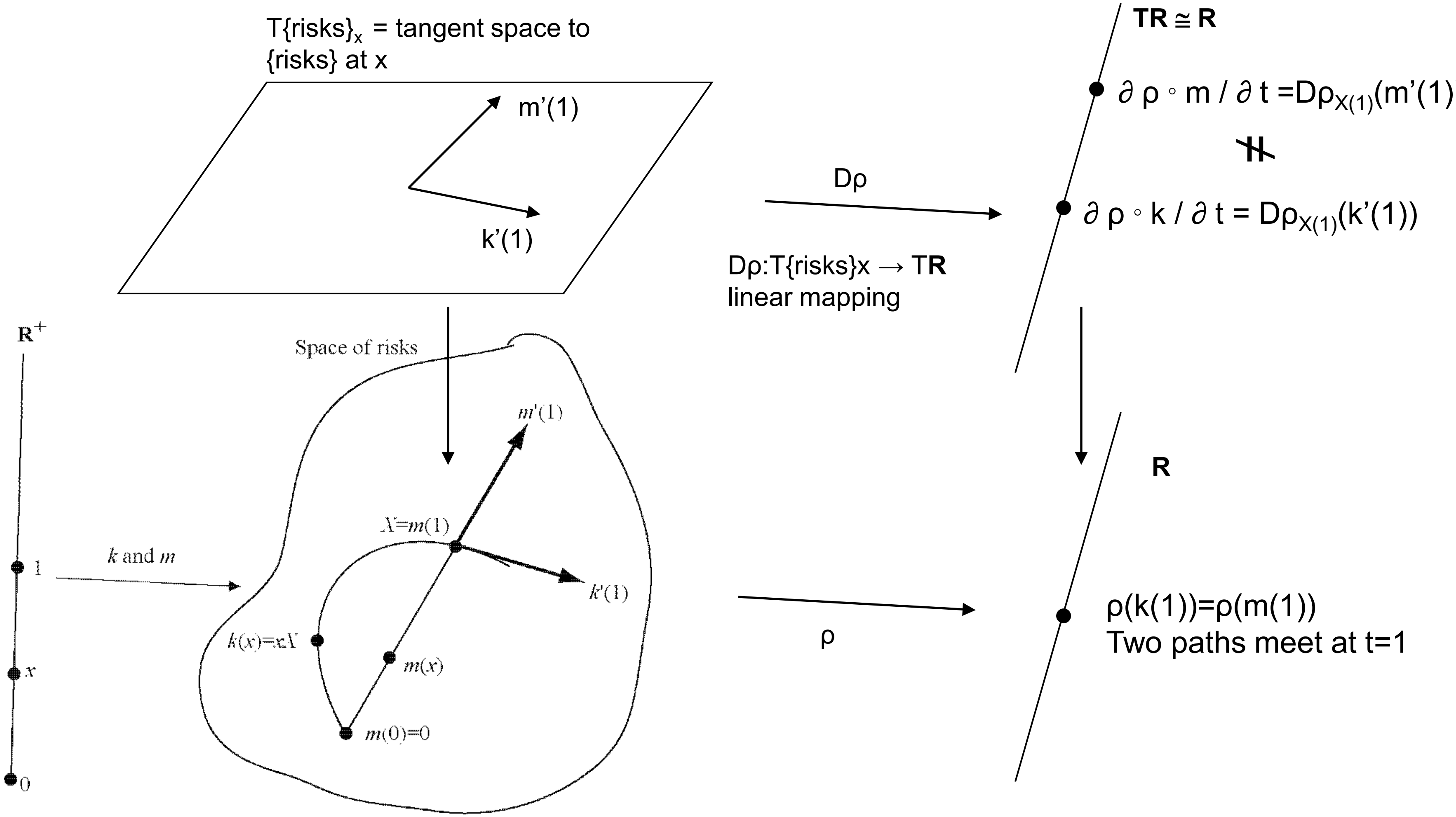
4. Homogeneity is not "Locally" Appropriate



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4. Homogeneity is not “Locally” Appropriate

- Why is m drawn as the straight line or ray (half-line)?

Table 1: Possible characterizations of a ray in $\mathbb{R}^n$

Characterization of ray	Required structure on $\mathbb{R}^n$
α is the shortest distance between $\alpha(0)$ and $\alpha(1)$	Notion of distance in $\mathbb{R}^n$, differentiable manifold
$\alpha''(t) = 0$, constant velocity, no acceleration	Very complicated on a general manifold.
$\alpha(t) = t\mathbf{x}$, $\mathbf{x} \in \mathbb{R}^n$.	Vector space structure
$\alpha(s+t) = \alpha(s) + \alpha(t)$	Can add in domain and range, semigroup structure only.

Asset return model, vectors x represent return distributions

- What is the addition operator “+” in { risks }?
  - Assets: vector space structure with basis of return variables
    - 3X ok = own three stocks etc.
  - Insurance: convolution of random variables
    - 3X not ok = over-insurance
    - $X_1 + X_2 + X_3$ ok using convolution sum of distributions

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4. Homogeneity is not "Locally" Appropriate

- Defining property for straight-line in { risks }: let X_t have distribution m(t) so $Pr(X_t<x) = m((- ,x])$
  - m(s + t) = m(s) ⋆ m(t), convolution sum of random variables
- Levy process satisfies m(s + t) = m(s) ⋆ m(t) and so is **appropriate notion of a straight line**
  - Additive, independent, homogeneous increments, stochastically continuous
- Examples of Levy processes
  - Brownian motion, compound Poisson, drift, combinations
- What are k'(1) and m'(1)?
  - m_t defines a family of probability measures
  - Properties manifest through **operator action** on functions $< f, m_t > = \int f(x)\, dm_t(x) = E(f(X_t))$
  - Derivative should be a **family of linear functionals** $f \to m_t'(f)$ indexed by t
  - Fundamental Theorem of Calculus: $< f, m(1) > - < f, m(0) > = \int_0^1 m_t'(f)\, dt$
  - Differentially: let $A(f)(x) = \lim_{s\to 0+} [\, E(f(x+X_s) - f(x))\,] / s$, then the derivative operator $f \to m_t'(f)$ satisfies $m_t'(f) = < Af, m_t >$

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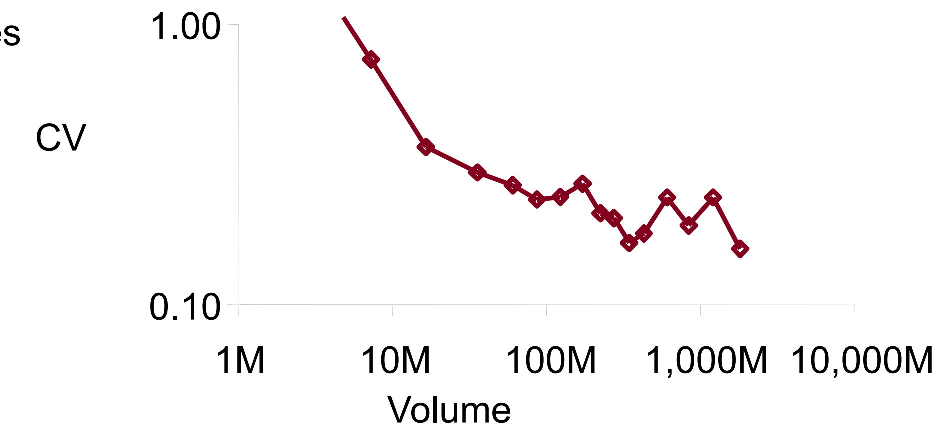
4. Homogeneity is not "Locally" Appropriate

- $A(f)(x) := \lim_{s\to 0+} [\, E(f(x+X_s) - f(x))\,] / s$ defines the infinitesimal generator A of the Markov process X_t
- For compound Poisson m, let J be distribution of jump sizes (severity)
- For small t, Pr(jump) = $\lambda t + O(t^2)$, so, conditioning on presence of a jump,
  - $A(f)(x) = \lim_{s\to 0+} [\, E(f(x+X_s)) - f(x))\,] / s = [\, \lambda s\, E(f(x+J)) + (1 - \lambda s)\, f(x) - f(x)\,] / s = \lambda\, (E(f(x+J)) - f(x))$

  and hence
  - **$m_t'(f) = < Af, m_t > = \lambda\, E[\, f(X_t+J) - f(X_t)$**], note expectation over independent variables X_t and J
- For k embedding k(t) = tX, A(f)(x) = E(X)f'(x) and so
  - **$k_t'(f) = < Af, m_t > = E[\, E(X)\, f'(tX)\,] = E(X)E(f'(tX))$**, which is completely different
- If J=1 is constant, so X_t = Poisson(λt) and $k(t) = kX_1$
  - $m_t'(f) = \lambda\, E[\, f(X_t + 1)) - f(X_t)\,]$
  - $k_t'(f) = E(X)\, E(f'(tX)) = \lambda\, E(\, f'(tX)\,)$
- If X_t includes a Brownian motion then A becomes a second order differential operator using Ito calculus
- Finally, $\lambda\, E[\, f(X_t+J) - f(X_t)\,]$ is a plausible risk measure for different functions f

5. Empirical Evidence

- We have seen the data supports two hypotheses
  - Risk is not homogeneous: i.e. CV not constant wrt volume
  - Risk is not volumetrically diversifying: CV has asymptote > 0



- Can we say more?

- Levy process based models, let X(.) be a Levy process
  - A(x,t) = X(xt) — volumetric/temporal symmetry
  - A(x,t) = X(xZ(t)) — Z a positive, increasing Levy process (a subordinator), e.g. seasons
  - A(x,t) = X(xCt) — E(C)=1, C is called a **mixing variable** (Heckman-Meyers)
  - A(x,t) = X(xCZ(t))

- The mixing variable appears unobservable, but can actually be derived from empirical data

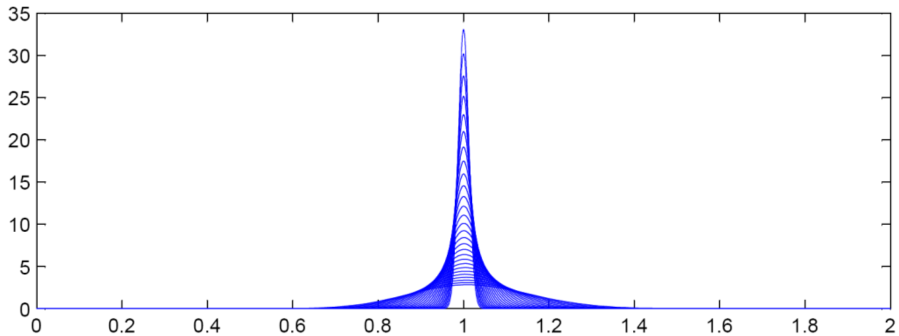
- Tame severity distributions are irrelevant

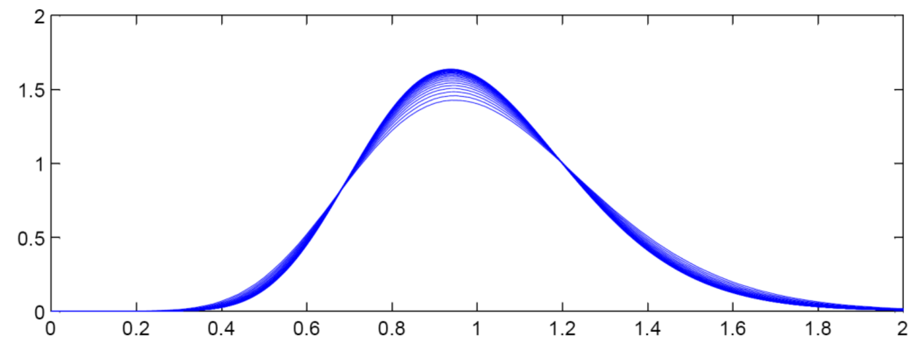
Mixing Variables & the Distribution of Normalized Loss Ratios

- Mixed compound Poisson: $A = X_1 + \ldots + X_N$, $N|C \sim \text{Poisson}(nC)$, $E(C)=1$
- Normalized Loss Ratio NLR = A / E(A)
- Dichotomous behavior of normalized loss ratios

No parameter uncertainty: leads to unrealistic aggregate loss distribution as expected losses increase

Including parameter preserves actual variability observed in data for large insurers





If C is constant, NLR converges to 1.0 in distribution

Illustration shows aggregates with Poisson frequency and larger & larger values of E(A)

If C is not constant, NLR converges to C in distribution

Illustration shows aggregates with negative binomial frequency (gamma mixing) & larger & larger values of E(A)

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Key Technical Result

- If severity X has a variance then A / E(A) converges in distribution to C as expected claim count tends to infinity

- Proof:

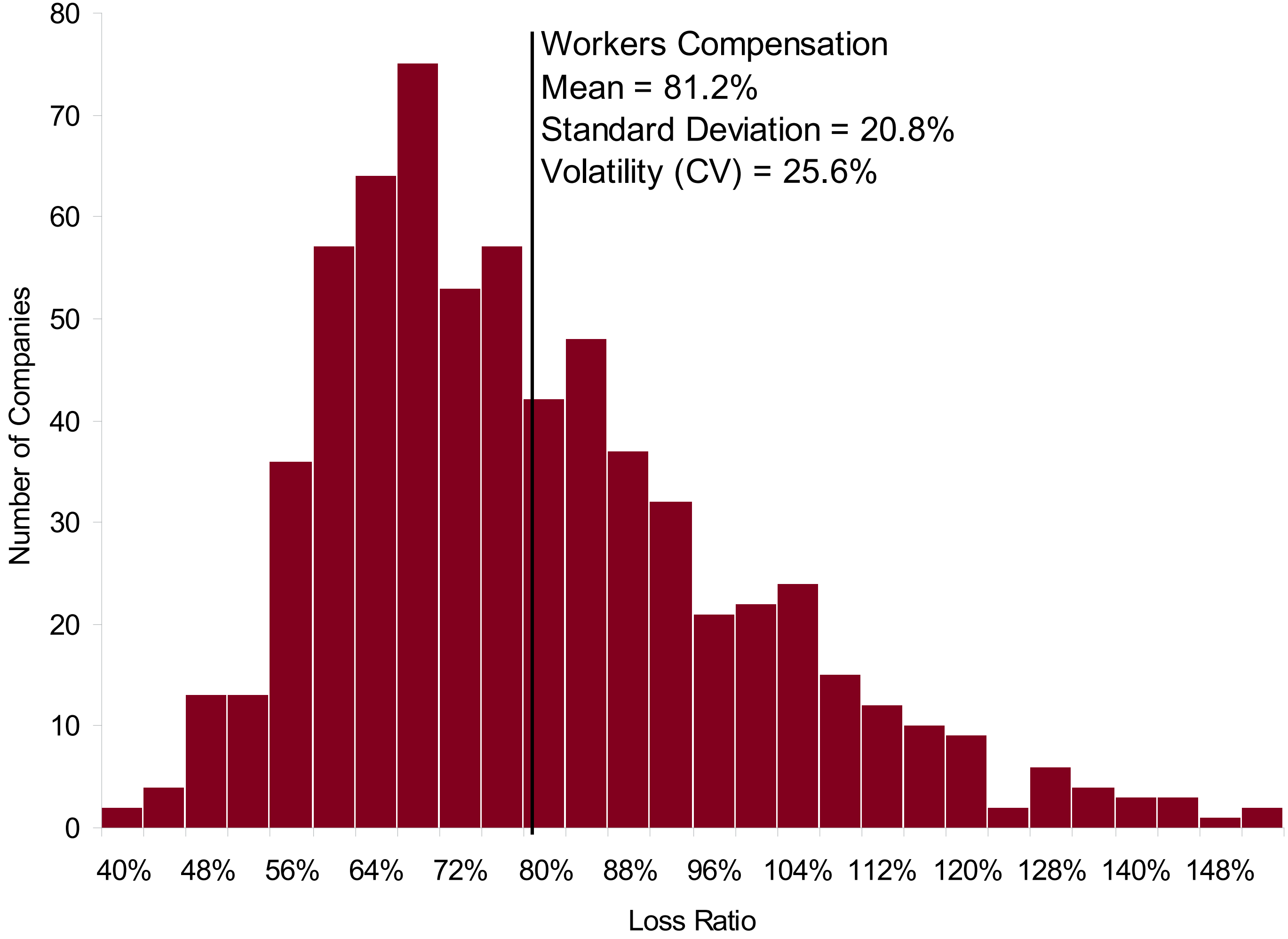
  Let M_D be the moment generating function $M_D(t)$ = E(exp(tD)) of D, for D=A, C, N or X. Let x=E(X), n=E(N), a=E(A)=nx. Then

$$\begin{aligned}
\lim_{n\to\infty} M_{A/a}(t) &= \lim_{n\to\infty} M_A(t/a) \\
&= \lim_{n\to\infty} M_C(n(M_X(t/a)-1)) \\
&= \lim_{n\to\infty} M_G(n(M'_X(0)t/nx + R(t/nx))) \\
&= \lim_{n\to\infty} M_C(t + nR(t/nx)) \\
&= M_C(t)
\end{aligned}$$

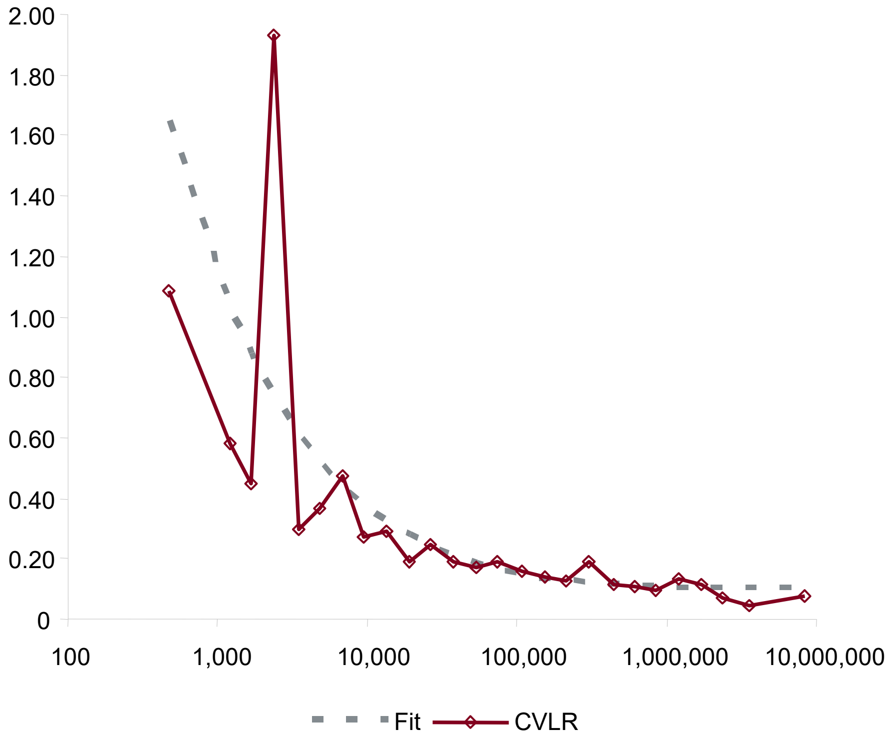
  For some remainder function $R(t)=O(t^2)$. The assumptions on X guarantee that $M_X'(0)=x=E(X)$ & that the reminder term in Taylor's expansion is $O(t^2)$. The result follows because a distribution is uniquely determined by its moment generating function.

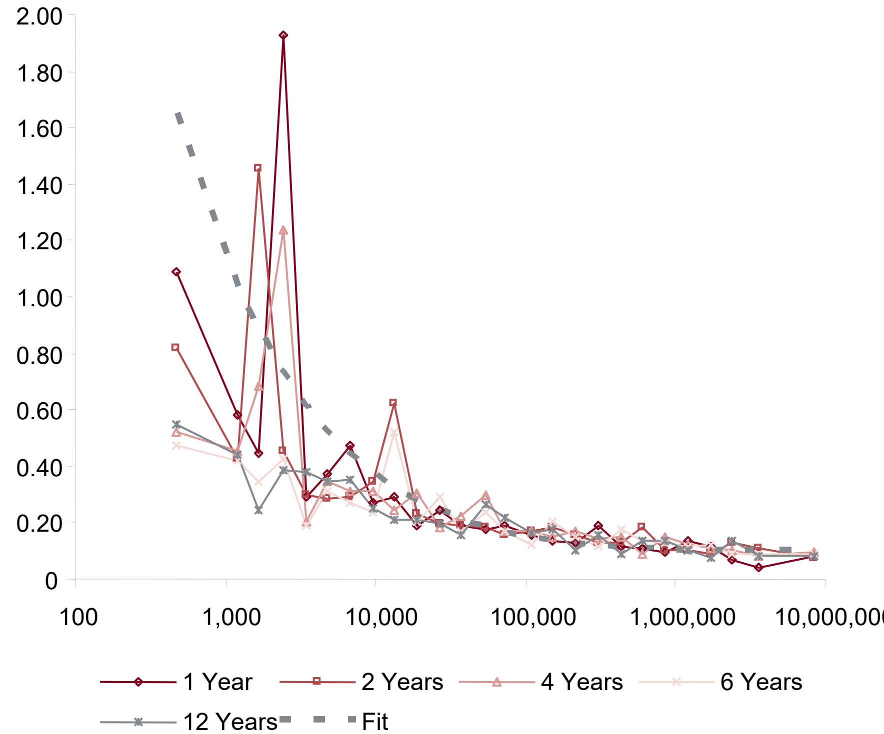
- Result has important implications for parameterizing economic capital simulation models and for understanding correlation between different lines of business

5. Empirical Evidence: Systemic Insurance Risk by Line



5. Empirical Evidence: Volumetric/Temporal Symmetry





- Consider volatility of A(x,t), A(2x,t/2), A(4x,t/4) etc.
- Same relationship between volatility and volume, xt
- **Data consistent with volumetric/temporal symmetry and with model A(x,t) = X(xCt)**

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6. Four Levy Process Models

- A(x,t) = X(xt) no Volumetrically diversifying
- A(x,t) = X(xZ(t)) no Volumetric/temporal asymmetry
- **A(x,t) = X(xCt) Yes Not volumetrically diversifying, volumetric/temporal symmetry**
- A(x,t) = X(xCZ(t)) no Volumetric/temporal asymmetry
- A(x,t) = xR(t) no Constant volatility with volume

Model	Variance	$v(x,t)$	Diversifying $x \to \infty$	Diversifying $t \to \infty$
$X(xt)$	$\sigma^2 xt$	$\frac{\sigma}{\sqrt{xt}}$	Yes	Yes
$X(xZ(t))$	$xt(\sigma^2 + x\tau^2)$	$\sqrt{\frac{\sigma^2}{xt} + \frac{\tau^2}{t}}$	No	Yes
$X(xCt)$	$xt(\sigma^2 + cxt)$	$\sqrt{\frac{\sigma^2}{xt} + c}$	No	No
$X(xCZ(t))$	$x^2t^2\left(\frac{(c+1)\tau^2}{t} + c\right) + \sigma^2 xt$	$\sqrt{\frac{\sigma^2}{xt} + \frac{\tau'^2}{t} + c}$	No	No
$xX(t)$	$x^2\sigma^2 t$	$\sigma/\sqrt{t}$	Const.	Yes

Variance and coefficient of variation v of each model

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7. Why bother with Levy Processes?

- Paper uses compound Poisson distributions as examples for simplicity
- Why bother with general Levy processes?
  - Academically interesting / publishing cottage industry!
- "Infinite activity" Levy processes include processes with X(1) distributed as
  - Lognormal
  - Pareto
  - Gamma
  - Laplace
  - Weibull ($\alpha<1$; $\alpha>1$ is not infinitely divisible)
  - Allows for negative jumps but positive creep
- Use of infinitesimal generator as a risk measure through norm of operator appears interesting

8. So What? Can we see the Impact in Prices?

- Idiosyncratic risk matters, price should decrease with size
  - Price = margin or spread over actuarial rate
  - Size = expected loss = xt; t=1
  - “Large” depends on particulars of severity distribution

- Umbrella and high limit policies
  - Companies target higher price and lower combined ratio for higher process risk

- Reinsurer notion of “balance”
  - Unbalanced cover has premium < limit

- Property per risk reinsurance
  - Large limits; unbalanced
  - Historically very expensive

- Large accounts, package policies
  - Probably top-line focus rather than risk theory

- Myers-Cohn
  - Impact of inhomogeneity apparent around volume typical of company business unit or allocation unit

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9. Observed Correlations and Copulas

US Statutory Loss Ratio Correlations

$100M Premium Threshold in Both Lines

First Year	1992	Evaluation	latest	Premium Threshold ($M)	100.0
Last Year	2007	Gross or Net	G	Averages	straight

Correlation Coefficients

Raw	Line B									
Line A	All	Home	PPAuto	CMP	CommAuto	WorkComp	OtherLiabOcc	MedMalCM	OtherLiabCM	ProdLiab-Occ
All	1.000	0.635	0.553	0.774	0.670	0.758	0.736	0.704	0.570	0.618
Home	0.635	**1.000**	**0.069**	0.198	0.079	-0.086	-0.021	-0.091	-0.043	0.102
PPAuto	0.553	**0.069**	**1.000**	0.250	0.281	0.305	0.295	0.314	0.366	0.270
CMP	0.774	0.198	0.250	**1.000**	**0.528**	**0.432**	**0.503**	0.595	0.423	0.427
CommAuto	0.670	0.079	0.281	**0.528**	**1.000**	**0.627**	**0.685**	0.725	0.451	0.752
WorkComp	0.758	-0.086	0.305	**0.432**	**0.627**	**1.000**	**0.638**	0.759	0.572	0.605
OtherLiabOcc	0.736	-0.021	0.295	**0.503**	**0.685**	**0.638**	**1.000**	0.802	0.606	0.641
MedMalCM	0.704	-0.091	0.314	0.595	0.725	0.759	0.802	**1.000**	**0.731**	**0.797**
OtherLiabCM	0.570	-0.043	0.366	0.423	0.451	0.572	0.606	**0.731**	**1.000**	**0.229**
ProdLiab-Occ	0.618	0.102	0.270	0.427	0.752	0.605	0.641	**0.797**	**0.229**	**1.000**
No Market Risk	Line B									
Line A	All	Home	PPAuto	CMP	CommAuto	WorkComp	OtherLiabOcc	MedMalCM	OtherLiabCM	ProdLiab-Occ
All	1.000	0.645	0.462	0.649	0.420	0.547	0.545	0.567	0.288	0.368
Home	0.645	**1.000**	**0.071**	0.083	0.080	-0.098	0.002	-0.167	0.068	0.123
PPAuto	0.462	**0.071**	**1.000**	0.082	0.047	0.143	0.107	0.155	0.107	0.372
CMP	0.649	0.083	0.082	**1.000**	**0.321**	**0.281**	**0.285**	0.046	0.158	0.222
CommAuto	0.420	0.080	0.047	**0.321**	**1.000**	**0.394**	**0.440**	0.273	0.128	0.371
WorkComp	0.547	-0.098	0.143	**0.281**	**0.394**	**1.000**	**0.226**	0.316	0.005	0.386
OtherLiabOcc	0.545	0.002	0.107	**0.285**	**0.440**	**0.226**	**1.000**	0.377	0.251	0.371
MedMalCM	0.567	-0.167	0.155	0.046	0.273	0.316	0.377	**1.000**	**0.426**	**0.206**
OtherLiabCM	0.288	0.068	0.107	0.158	0.128	0.005	0.251	**0.426**	**1.000**	**0.099**
ProdLiab-Occ	0.368	0.123	0.372	0.222	0.371	0.386	0.371	**0.206**	**0.099**	**1.000**

Numbers in gray not statistically significantly different from zero at 90% level

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9. Observed Correlations and Copulas

First Year	1992	Evaluation	latest	Premium Threshold ($M)	100.0
Last Year	2007	Gross or Net	G	Averages	straight

90.0% Confidence Interval for Correlation Coefficients

Raw	Line B									
Line A	All	Home	PPAuto	CMP	CommAuto	WorkComp	OtherLiabOcc	MedMalCM	OtherLiabCM	ProdLiab-Occ
All	1	(0.60, 0.67)	(0.52, 0.58)	(0.75, 0.80)	(0.63, 0.70)	(0.73, 0.78)	(0.71, 0.76)	(0.65, 0.75)	(0.50, 0.63)	(0.51, 0.71)
Home	(0.60, 0.67)	1	(0.01, 0.13)	(0.12, 0.27)	(-0.01, 0.16)	(-0.17, 0.00)	(-0.11, 0.07)	(-0.32, 0.14)	(-0.18, 0.09)	(-0.09, 0.29)
PPAuto	(0.52, 0.58)	(0.01, 0.13)	1	(0.18, 0.32)	(0.21, 0.35)	(0.23, 0.37)	(0.22, 0.37)	(0.11, 0.50)	(0.25, 0.47)	(0.09, 0.44)
CMP	(0.75, 0.80)	(0.12, 0.27)	(0.18, 0.32)	1	(0.47, 0.58)	(0.37, 0.49)	(0.44, 0.56)	(0.46, 0.71)	(0.33, 0.51)	(0.28, 0.56)
CommAuto	(0.63, 0.70)	(-0.01, 0.16)	(0.21, 0.35)	(0.47, 0.58)	1	(0.58, 0.67)	(0.64, 0.72)	(0.62, 0.81)	(0.36, 0.54)	(0.67, 0.82)
WorkComp	(0.73, 0.78)	(-0.17, 0.00)	(0.23, 0.37)	(0.37, 0.49)	(0.58, 0.67)	1	(0.59, 0.68)	(0.67, 0.83)	(0.49, 0.64)	(0.49, 0.70)
OtherLiabOcc	(0.71, 0.76)	(-0.11, 0.07)	(0.22, 0.37)	(0.44, 0.56)	(0.64, 0.72)	(0.59, 0.68)	1	(0.73, 0.86)	(0.54, 0.67)	(0.53, 0.73)
MedMalCM	(0.65, 0.75)	(-0.32, 0.14)	(0.11, 0.50)	(0.46, 0.71)	(0.62, 0.81)	(0.67, 0.83)	(0.73, 0.86)	1	(0.64, 0.81)	(0.68, 0.88)
OtherLiabCM	(0.50, 0.63)	(-0.18, 0.09)	(0.25, 0.47)	(0.33, 0.51)	(0.36, 0.54)	(0.49, 0.64)	(0.54, 0.67)	(0.64, 0.81)	1	(0.05, 0.39)
ProdLiab-Occ	(0.51, 0.71)	(-0.09, 0.29)	(0.09, 0.44)	(0.28, 0.56)	(0.67, 0.82)	(0.49, 0.70)	(0.53, 0.73)	(0.68, 0.88)	(0.05, 0.39)	1
No Market Risk	Line B									
Line A	All	Home	PPAuto	CMP	CommAuto	WorkComp	OtherLiabOcc	MedMalCM	OtherLiabCM	ProdLiab-Occ
All	1	(0.61, 0.68)	(0.42, 0.50)	(0.61, 0.68)	(0.37, 0.47)	(0.51, 0.58)	(0.50, 0.59)	(0.49, 0.63)	(0.20, 0.37)	(0.22, 0.50)
Home	(0.61, 0.68)	1	(0.01, 0.13)	(0.00, 0.16)	(0.00, 0.16)	(-0.18, -0.01)	(-0.09, 0.10)	(-0.38, 0.06)	(-0.07, 0.20)	(-0.07, 0.31)
PPAuto	(0.42, 0.50)	(0.01, 0.13)	1	(0.01, 0.16)	(-0.03, 0.12)	(0.07, 0.22)	(0.02, 0.19)	(-0.06, 0.36)	(-0.02, 0.23)	(0.20, 0.53)
CMP	(0.61, 0.68)	(0.00, 0.16)	(0.01, 0.16)	1	(0.25, 0.39)	(0.21, 0.35)	(0.21, 0.36)	(-0.14, 0.23)	(0.05, 0.26)	(0.06, 0.38)
CommAuto	(0.37, 0.47)	(0.00, 0.16)	(-0.03, 0.12)	(0.25, 0.39)	1	(0.33, 0.45)	(0.38, 0.50)	(0.09, 0.44)	(0.01, 0.24)	(0.22, 0.51)
WorkComp	(0.51, 0.58)	(-0.18, -0.01)	(0.07, 0.22)	(0.21, 0.35)	(0.33, 0.45)	1	(0.15, 0.30)	(0.14, 0.48)	(-0.11, 0.12)	(0.24, 0.52)
OtherLiabOcc	(0.50, 0.59)	(-0.09, 0.10)	(0.02, 0.19)	(0.21, 0.36)	(0.38, 0.50)	(0.15, 0.30)	1	(0.22, 0.52)	(0.15, 0.35)	(0.22, 0.51)
MedMalCM	(0.49, 0.63)	(-0.38, 0.06)	(-0.06, 0.36)	(-0.14, 0.23)	(0.09, 0.44)	(0.14, 0.48)	(0.22, 0.52)	1	(0.27, 0.56)	(-0.06, 0.44)
OtherLiabCM	(0.20, 0.37)	(-0.07, 0.20)	(-0.02, 0.23)	(0.05, 0.26)	(0.01, 0.24)	(-0.11, 0.12)	(0.15, 0.35)	(0.27, 0.56)	1	(-0.08, 0.27)
ProdLiab-Occ	(0.22, 0.50)	(-0.07, 0.31)	(0.20, 0.53)	(0.06, 0.38)	(0.22, 0.51)	(0.24, 0.52)	(0.22, 0.51)	(-0.06, 0.44)	(-0.08, 0.27)	1

Number of Observations

Line A	All	Home	PPAuto	CMP	CommAuto	WorkComp	OtherLiabOcc	MedMalCM	OtherLiabCM	ProdLiab-Occ
All	4400	852	1260	702	671	1022	653	248	296	99
Home	852	852	722	423	388	378	308	52	144	73
PPAuto	1260	722	1260	453	483	455	376	61	167	77
CMP	702	423	453	702	488	516	435	79	222	97
CommAuto	671	388	483	488	671	543	464	77	204	98
WorkComp	1022	378	455	516	543	1022	477	80	221	99
OtherLiabOcc	653	308	376	435	464	477	653	88	249	98
MedMalCM	248	52	61	79	77	80	88	248	87	41
OtherLiabCM	296	144	167	222	204	221	249	87	296	87
ProdLiab-Occ	99	73	77	97	98	99	98	41	87	99

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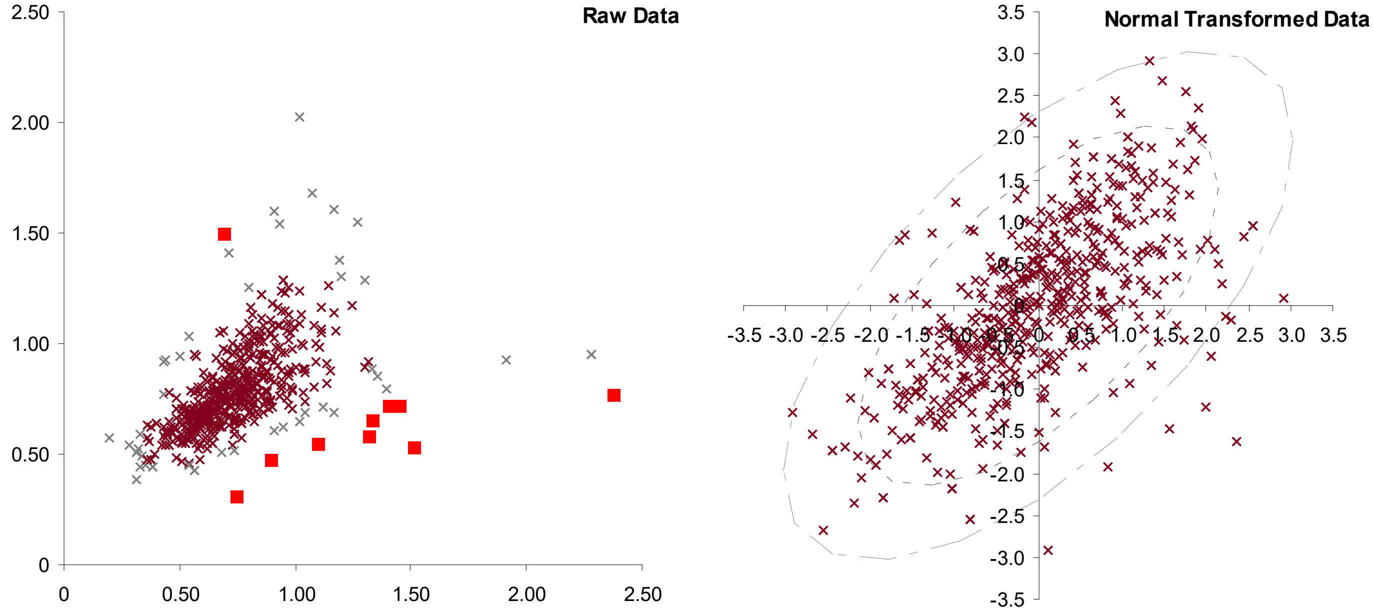
9. Observed Correlations and Copulas

Commercial Multi-Peril (x-axis) vs. Commercial Auto Liability, $100M premium threshold **552 Annual Observations**



Association Summary

Linear Correlation, rho	52.1%
90% Confidence Interval	(46.8%, 57.1%)
Base Linear Correlation	71.1%
Extreme Linear Correlation (n=57)	30.9%
Rank Correlation	67.6%
Rank Correlation from rho	50.3%
Normal-Transformed Correlation	65.1%
Kendall Tau	50.0%
Rho from tau	70.7%
Outliers at 10% and 1% levels	10.3% and 1.8%

Univariate Summary

	Commercial Multi-Peril	Commercial Auto Liabilit
Mean	0.7448	0.7865
Min	0.1961	0.3119
Max	2.3773	2.0247
Std. Dev.	0.2409	0.2050
CV	32.3%	26.1%
Skewness	1.99	1.36
Kurtosis	9.88	3.91
90th percentile	98.7%	105.1%
99th percentile	142.8%	151.9%

Note: 1% outliers from normal copula marked in red. 10% and 1% and confidence intervals show on right.

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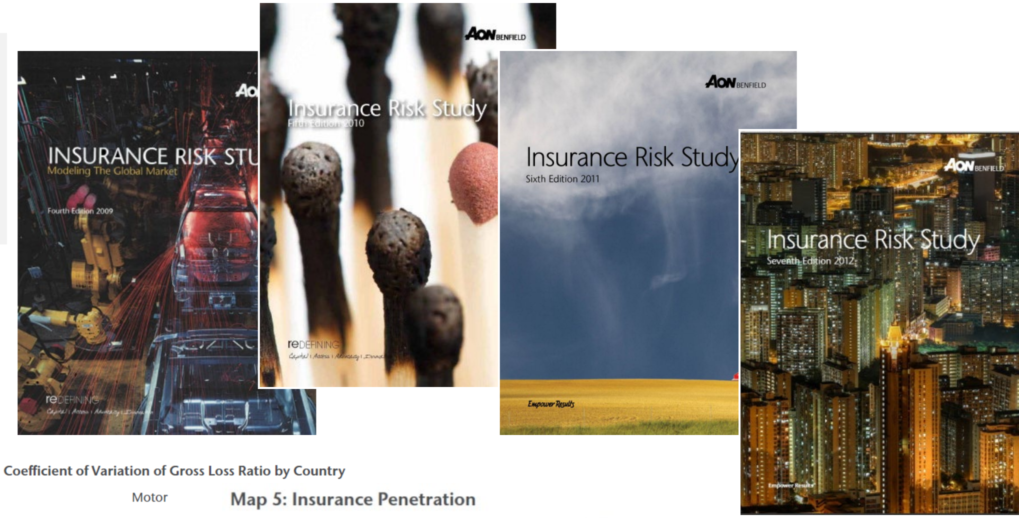
10. How the Results are Used

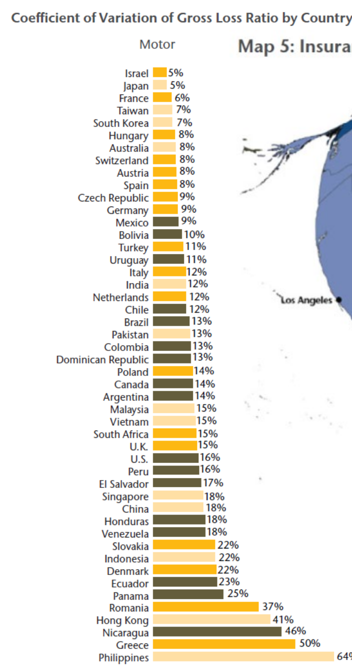
Aon Benfield Insurance Risk Study Informed Parameterization of Risk Models

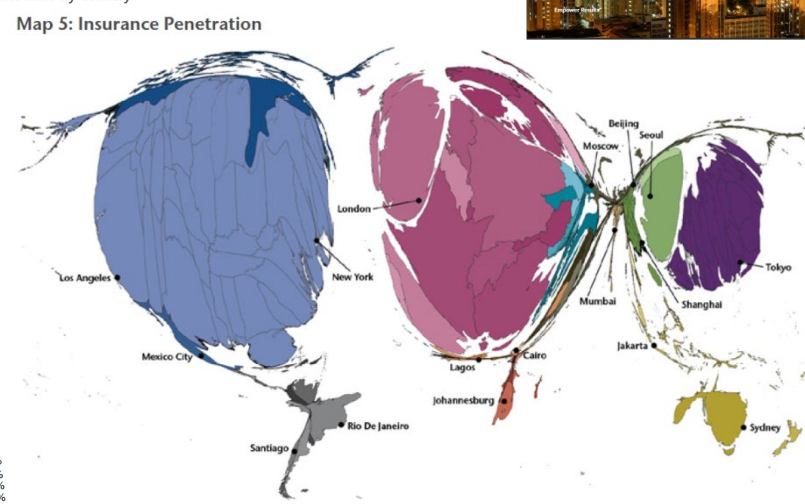
Objective

- Insurance Risk Study determines credible global insurance volatility benchmarks for use in underwriting risk modeling

- Motivation: robust empirical quantification of all aspects of underwriting risk
- Systemic volatility parameters by country, by line
  - Forty eight countries, 90% of global premium
  - Results for eight core lines of business
  - Available as input to any simulation tool
- Loss ratio correlation between lines within country and between countries
- Assessment of US reserve risk
- Correlation between macroeconomic and insurance variables
- Economic and insured loss potential from major catastrophe risks globally
- Recognized by major US rating agencies
- Published annually in August
- Seventh edition released in 2012







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Contact Information

Stephen J. Mildenhall, PhD, FCAS, ASA, CERA

Aon Benfield Analytics
Chicago, IL

+1.312.381.5880 (office) / +1.312.961.8781 (cell)
stephen.mildenhall@aonbenfield.com